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Global Dynamic of Prey-Predator Model with General Incidence

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الديناميكية الشاملة لنموذج الفريسة والمفترس عند الدالة المعممة H(S,I)

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Abstract:
In this paper, we consider a prey-predator system in which the susceptible and infected prey populations are predated by a predator species. We also discuss different models of prey-predator systems with a general incidence rate H(S, I). The aim of this paper is to study the dynamics of the prey-predator model using different analytical techniques with a generalized incidence function, and to investigate the nature of its equilibria and global stability properties.

Keywords: Prey-predator, Equilibrium, Endemic, Stability, Lyapunov.

الملخص
نتناول في هذه الورقة البحثية نظام الفريسة والمفترس حيث تتعرض عشائر الفرائس القابلة للإصابة والمصابة للافتراس من قِبل نوع مفترس. كما نناقش نماذج مختلفة لأنظمة الفريسة والمفترس بمعدل حدوث عام H(S, I). يهدف هذا البحث إلى دراسة السلوك الديناميكي لنموذج الفريسة والمفترس باستخدام تقنيات رياضية مختلفة في ظل دالة حدوث معممة، بالإضافة إلى دراسة طبيعة نقاط الاتزان وخصائص الاستقرار الشامل للنظام.

الكلمات المفتاحية: الفريسة والمفترس، نقطة الاتزان، الاستقرار، دالة ليابونوف.

1. Introduction

Over the past decades, the dynamical behavior of prey-predator models has received considerable attention. Mukherjee [1] showed global stability of a population in a polluted environment. Also, many researchers have looked at specific problems that can be solved using several models of prey-predator systems.

We present the following model:

$$\begin{aligned} \frac{dS}{dt} &= \theta - IH(S, I) - d_1 S \\ \frac{dI}{dt} &= IH(S, I) - \alpha IY - d_2 I \qquad (1) \\ \frac{dy}{dt} &= e\alpha Iy - d_3 y \end{aligned}$$

Where:
H(S, I): General incidence rate.

θ: Constant recruitment rate of susceptible prey.

α: Predation rate among the susceptible and infected prey population.

d_1, d_2, d_3: Natural death rates of the respective populations; e: the coefficient of biomass conversion adopted by the predator.

S(t), I(t), Y(t): Susceptible prey, infected prey, and predator populations at time t, respectively.

The dynamic of (1.1) in D is equivalent $D=\{(S,I,Y)\in R^3 : 0\leq S+I+Y\leq \frac{\theta}{d_1}\}$

$\zeta=\{(S,I,Y)\in R^3 : S+I+Y\leq \frac{\theta}{d_1}\}$

Theorem 1.

We define the quarantine reproduction number in the form

$$R_q = \frac{\frac{\theta}{d_1}}{\alpha y + d_2}$$

If $Rq \leq 1$, there exist unique equilibrium $P_0 = \left(\frac{\theta}{d_1}, 0,0\right)$ $if\ R_q \geq 1$

Then there exist an endemic non trivial equilibrium point

$$P^* = (S^*, I^*, Y^*) \in \zeta$$

Proof. The uniqueness of the endemic nontrivial equilibrium point can be guaranteed by [2]. From the isocline equations, it is clear that coordinates of the endemic equilibrium point $P^* = (S^*, I^*, Y^*)$ satisfy (1) . Hence since S^*exist, then H must be less than $\frac{\theta}{d_1}$, i.e. when $R_q > 1$, S^*does not exist and the only equilibrium points $P_0 = \left(\frac{\theta}{d_1}, 0,0\right)$. now the local stability of P_0can easily deduced by inspection of the eigenvalues of the following Jacobian matrix at P_0.

$$M_{P_0} = \left[-d_1 \ -H\left(\frac{\theta}{d_1}, 0\right) \ 0 \ 0 \ H\left(\frac{\theta}{d_1}, 0\right) - d_2 \ 0 \ 0 \ 0 \ -d_3 \right]$$

Which has the eigenvalues $\lambda_1 = -d_1,\ \lambda_2 = H\left(\frac{\theta}{d_1}, 0\right) - d_2\ ,\ \lambda_3 = -d_3$

This completes the proof

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2. The Nature of Equilibria

In this section we study and discuss stability properties of the equilibrium .

The Jacobian matrix of (1.1) around E_0 is

$$[\ I\ H_2(S,I) - d_2 - \lambda \ -IH_I(S,I) + H(S,I)\ 0\ IH_S(S,I)\ IH_I(S,I) + H(S,I) - \alpha - d_2 - \lambda \ -\alpha I\ 0\ e\alpha y\ e\alpha I - d_3 - \lambda\ ] \ (2)$$

The dynamical behavior of model (1.1) is as follows :

$(a) E_0\left(\frac{\theta}{d_1}, 0,0\right)$ is non trivial equilibrium point .

(b) the eigenvalues corresponding to the equilibrium point $E_0\left(\frac{\theta}{d_1}, 0,0\right)$ are $-d_1, -\alpha + H(S^*, I^*) - d_2\ ,\ -d_3$ (2.2)

Thus it will be locally stable If $H(S^*, I^*) < d_2$ and $d_3 > 0$. Further this is always stable in (S) direction . It shows an unstable saddle point for $H(S^*, I^*) < d_2$ in (S), (I) direction

(c) The equilibrium point E_1 will biologically exist if $H(S^*, I^*) < d_2$

(d) The equilibrium point $E_2\ \left(0,0,\frac{d_1 d_3}{e\alpha_2 y}\right) d_3 > e\alpha I$

The variation matrix (2.1) for this position provides the eigenvalues

$$\lambda_2 = I^* H_1(S^*, I^*) + H(S^*, I^*) - \alpha y^* - d_2$$

And the two eigenvalues as the roots of the quadratic equation

$$\lambda^2 + \left(\alpha \underline{y} + d_1\right)\lambda = 0 \ (3)$$

(e) non zero equilibrium $E_3(S^*, I^*, y^*)$ is feasible provided (see[2])

$$e\alpha d_2 > H(S^*, I^*) d_3 + e\alpha_2 d_1$$

And

$$\frac{H(S^*,I^*)}{S^*}d_3 > \frac{H(S^*,I^*)}{S^*}e\alpha_2 I^* \quad (2.4)$$

Also

$$S^* = \frac{d_3 - e\alpha_2 I^*}{e\alpha_2 d_2} > 0$$

$$I^* = \frac{d_3}{e\alpha_2} - \frac{e\alpha_2 I^*}{e\alpha_2 \frac{d_3}{S} - e\alpha_2 d_2}$$

and

$$y^* = \frac{\frac{d_3}{S^*} - e\alpha\frac{I^*}{S^*}}{e\alpha I^*} > 0$$

3. Existence of Stability of E(S*, I*, Y*) Using Lyapunov Function

Let us consider Lyapunov function :

$$V(S,I,Y) = \sum_{i=1}^{2} \quad W_i\left(S_i - S_i^* - S_i^* \, log \, log \, \frac{S_i}{S_i^*}\right) + W_3\left(y_i - y_i^* - y_i^* \, log \, log \, \frac{y}{y_i^*}\right) \quad (4)$$

The time derivative of above function will be ……

$$\dot{V} = (t)W_1 Z_1 \frac{\dot{S}}{S} + W_2 Z_2 \frac{\dot{I}}{I} + W_3 Z \frac{\dot{y}}{y} \quad \ldots\ldots\ldots (5)$$

Where

$$Z_1 = \left(S - \dot{S}\right), Z_2 = \left(I - \dot{I}\right), Z_3 = \left(Y - \dot{Y}\right)$$

Using the set of equations (1.1) and (3.2) we obtain

$$\dot{V}(t) = -\frac{\theta W_1 Z_1^2}{S\dot{S}} - \frac{H(S,I)}{S}W_1 Z_1 Z_2 - \alpha W_1 Z_1 Z_3 + \frac{H(S,I)}{S}W_2 Z_1 Z_2 - \alpha W_2 Z_2 Z_3 + e\alpha W_3 Z_2 Z_3$$

we can choose $W_1 = W_2$ and $W_3 = \frac{1}{e}$ we get

$$\dot{V}(t) = -\frac{\theta Z_1^2}{S\dot{S}} \leq 0$$

This shows that $E_3$is globally asymptotically stable . Hence Lyapunov theorem implies that $E_3(S^*, I^*, Y^*)$ is globally asymptotically stable [3, 4]

4. Conclusion

Our main results are concerned for discussing equilibria stability existence and uniqueness with general incidence H(S,I) when

E_0 is non trivial equilibria and unique endemic point.

$P_0 = \left(\frac{\theta}{d_1}, 0,0\right)$ is globally asymptotically stable in ζ if $R_q \geq 1$,

While its an unstable saddle point if $Rq \leq 1$.

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