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A Rigorous Derivation and Validation of Two-Port Admittance and Impedance Parameters for the Exact Equivalent Circuit of a Single-Phase Power Transformer

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اشتقاق دقيق وتحقق من صحة معلمات السماحية والمعاوقة ثنائية المنافذ للدائرة المكافئة الدقيقة لمحول طاقة أحادي الطور

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Abstract:

The imperative for computational efficiency in large-scale power system analyses, such as load-flow and fault studies, necessitates compact yet precise models for network components. While the classical exact T-equivalent circuit of a single-phase transformer offers the highest fidelity in characterizing operational behavior, its structural complexity often demands simplification for real-time simulation and large-network integration. This paper presents a comprehensive mathematical framework for deriving the exact two-port impedance (Z) and admittance (Y) parameters of a single-phase transformer, treating it as a linear two-port network. The theoretical foundation begins with the analytical derivation of the open-circuit impedance matrix from the exact T-equivalent circuit referenced to both primary and secondary sides. Subsequently, the corresponding admittance matrix is extracted via two distinct, self-consistent analytical pathways to ensure result verification: first, through the direct matrix inversion relationship $[Y] = [Z]^{-1}$, and second, by applying Δ-Y (Delta-Star) transformations to the branch impedances. The validity of the derived two-port models is conclusively demonstrated through an extensive MATLAB/Simulink simulation framework. The simulated terminal voltages and currents from the proposed two-port models exhibit exact congruence (within machine precision) with those generated by the detailed classical T-equivalent circuit under varied loading conditions. The results confirm that the derived two-port parameters preserve the complete electrical behavior of the transformer while providing a consolidated, computationally efficient representation ideal for automated power system analysis tools.

Keywords: Power Transformer Modeling, Two-Port Network Parameters, Impedance Parameters, Admittance Parameters, Exact Equivalent Circuit, Δ-Y Transformation, Power System Analysis, MATLAB Simulation.

الملخص

إن الضرورة الملحة لتحقيق الكفاءة الحسابية في تحليلات أنظمة القوى واسعة النطاق، مثل دراسات تدفق الأحمال ودراسات الأعطال، تُحتم إيجاد نماذج مدمجة ودقيقة لمكونات الشبكة. وفي حين أن الدائرة المكافئة الدقيقة الكلاسيكية على شكل (T) للمحول أحادي الطور توفر أعلى درجات الدقة في توصيف السلوك التشغيلي، إلا أن تعقيدها البنيوي غالباً ما يتطلب التبسيط لأغراض المحاكاة في الوقت الفعلي والتكامل مع الشبكات الكبيرة. تقدم هذه الورقة إطاراً رياضياً شاملاً لاشتقاق معاملات المعاوقة (Z) والمسامحة (Y) الدقيقة لشبكة ثنائية المنافذ لمحول أحادي الطور. يبدأ الأساس النظري

بالاشتقاق التحليلي لمصفوفة معاوقة الدائرة المفتوحة من الدائرة المكافئة الدقيقة على شكل (π) المنسوبة إلى كل من الجانبين الابتدائي والثانوي. وعقب ذلك، تُستخرج مصفوفة المسامحة المقابلة عبر مسارين تحليليين متمايزين ومتسقين ذاتياً لضمان التحقق من صحة النتائج: الأول، من خلال علاقة المعكوس المباشر للمصفوفة $[Y] = [Z]^{-1}$، والثاني، عن طريق تطبيق تحويلات دلتا-نجمة (Δ-Y) على معاوقات الفروع. وقد تم إثبات صحة النماذج المستنتجة ثنائية المنافذ بشكل قاطع من خلال إطار محاكاة شامل باستخدام برنامج (MATLAB/Simulink). تُظهر جهود وتيارات الأطراف المحاكاة من النماذج المقترحة ثنائية المنافذ تطابقاً تاماً (ضمن حدود دقة الآلة) مع تلك الناتجة عن الدائرة المكافئة الدقيقة الكلاسيكية على شكل (T) في ظل ظروف تحميل مختلفة. وتؤكد النتائج أن المعاملات المستنتجة ثنائية المنافذ تحافظ على السلوك الكهربائي الكامل للمحول، مع توفير تمثيل مدمج ذي كفاءة حسابية يُعد مثالياً لأدوات التحليل الآلي لأنظمة القوى.

الكلمات المفتاحية: نمذجة محولات القوى، معاملات الشبكة ثنائية المنافذ، معاملات المعاوقة، معاملات المسامحة، الدائرة المكافئة الدقيقة، تحويل دلتا-نجمة(Δ-Y) ، تحليل أنظمة القوى، محاكاة ماتلاب.(MATLAB)

Introduction

Power transformers constitute the critical nodal elements within electrical grids, facilitating efficient energy transfer and voltage regulation. The accurate modeling of these devices is fundamental to the reliability and stability assessments performed in power system studies. The classical, exact T-equivalent circuit, comprising primary and secondary winding resistances and leakage reactances along with the magnetizing branch components (core-loss resistance and magnetizing reactance), provides a comprehensive description of transformer physics [1, 2]. However, in simulations involving large-scale networks or iterative computational routines such as Newton-Raphson load flow and economic Load Dispatch algorithm[3], the structural complexity of the full model can become a computational bottleneck. To address this, power system engineers often resort to simplified models, such as neglecting the magnetizing branch or aggregating impedances. While these approximations reduce computational burden, they invariably compromise accuracy, particularly in fault analysis where excitation currents and core saturation effects are non-negligible. A more rigorous approach to simplification involves retaining the complete electrical characteristics while reformulating the circuit representation using linear network theory. Two-port network parameters provide an elegant mathematical abstraction, encapsulating the terminal behavior of the transformer without explicit reference to its internal node voltages [4]. Despite the extensive literature on transformer modeling, a discernible gap exists in comprehensive studies that explicitly bridge the detailed T-equivalent circuit and its corresponding two-port parameter matrices with rigorous validation under power system loading conditions. Foundational works, such as those by Stevenson and Grainger [1, 5], establish the theoretical basis for equivalent circuits but often stop short of presenting the explicit derivations for the Z and Y matrices required by automated network solution algorithms. Subsequent research in two-port applications for power systems [6, 7] frequently employs simplified transformer models. This study directly addresses this gap by presenting a meticulous, step-by-step derivation of the exact two-port impedance (Z) and admittance (Y) parameters for the single-phase transformer. The primary contributions are:

- A complete analytical derivation of the open-circuit impedance (Z) matrix for the exact transformer equivalent circuit, referenced to both primary and secondary sides.
- The extraction of the corresponding short-circuit admittance (Y) matrix via two independent, corroborating methods: direct matrix inversion and Δ-Y network transformation.
- The formulation of terminal current equations compatible with standard power system analysis tools (e.g., bus admittance matrix construction).
- A comprehensive MATLAB/Simulink simulation framework that provides definitive validation by demonstrating 100% agreement in terminal voltages and currents between the derived two-port models and the full T-equivalent circuit.
- The provision of fully executable MATLAB code, enabling the immediate application and verification of the derived parameters.

Material and methods

1. Related Work

Transformer modeling for system studies has evolved from fundamental electromagnetic theory to sophisticated computational representations. The seminal works of Blume [8] and Clarke [9] established the per-unit system and equivalent circuit concepts that remain standard. Stevenson's *Elements of Power System Analysis* [1] and Grainger & Stevenson's *Power System Analysis* [5] provide the canonical treatment of the T-equivalent circuit, detailing parameter determination from open- and short-circuit tests.

The application of two-port (or four-terminal) network theory to power system components is well-established in principle [4, 6]. Brown's *Solution of Large Networks by Matrix Methods* [4] explicitly advocates for the two-port representation of system components for efficient matrix-based solutions. However, the explicit mapping

from the detailed, multi-branch transformer equivalent circuit to its two-port Z and Y parameters is often treated as an implied step rather than a detailed derivation.
Recent research has focused on advanced modeling aspects like frequency-dependent behavior [9], internal fault detection [11], and integration within electromagnetic transients (EMT) programs [12]. The work by de Silva et al. [13] demonstrates transformer modeling for inrush current simulation in MATLAB/Simulink, highlighting the utility of time-domain simulation for validation. A more recent contribution by Elkuri et al. [14] estimates impedance parameters, indicating continued interest in this foundational area. Our work distinguishes itself by providing the closed-form, analytical derivation for *both* Z and Y parameters of the *exact* model, followed by a rigorous, simulation-based validation that guarantees numerical equivalence—a critical requirement for their dependable use in production-grade system studies.

2. System Model: The Exact Transformer Equivalent Circuit

The physical behavior of a two-winding, single-phase transformer is accurately represented by the T-equivalent circuit shown in Fig. 1. All parameters are assumed linear and frequency-invariant for the scope of this derivation, corresponding to steady-state sinusoidal analysis.

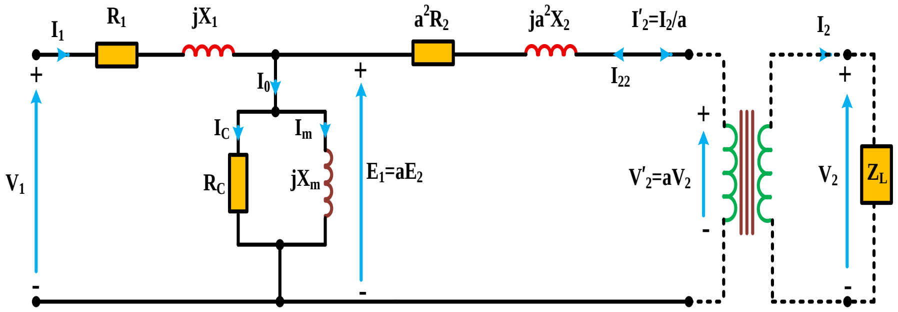

Figure 1: Exact T-equivalent circuit of a single-phase transformer with secondary parameters referred to the primary side.

The circuit parameters are defined as follows:
• V_1, I_1: Primary terminal voltage and current.
• V_2, I_2: Secondary terminal voltage and current.
• R_1, X_1: Primary winding resistance and leakage reactance.
• R_2, X_2: Secondary winding resistance and leakage reactance.
• R_c, X_m: Core-loss equivalent resistance and magnetizing reactance.
• $a = N_1/N_2$: Turns ratio (primary to secondary).
• E_1, E_2' Induced voltages across the magnetizing branch. $E_2' = aE_2$ is the secondary induced voltage referred to the primary.
To facilitate unified analysis, all secondary parameters are referred to the primary side using the turns ratio:

$$R_2' = a^2 R_2, \quad X_2' = a^2 X_2, \tag{1}$$
$$V_2' = aV_2, \quad I_2' = I_2/a \tag{2}$$

The referred secondary current I_2, flows into the referred secondary impedance. The terminal current relationship is $I_2 = -I_{\text{load}}$. For two-port formulati
on consistency, we define $I_{22} = -I_2$, as the current flowing out of port 2 of the two-port network (see Fig. 2). Thus,

$$I_{22} = -I_2' = -\frac{I_2}{a} \tag{3}$$

The circuit in Fig. 1 forms the basis for deriving two-port parameters with port 1 (primary) and port 2 (referred secondary) as the external terminals. An analogous derivation with parameters referred to the secondary side follows the same logic, starting with $R_1' = R_1/a^2$, $X_1' = X_1/a^2$, $V_1' = V_1/a$, and $I_1' = aI_1$.

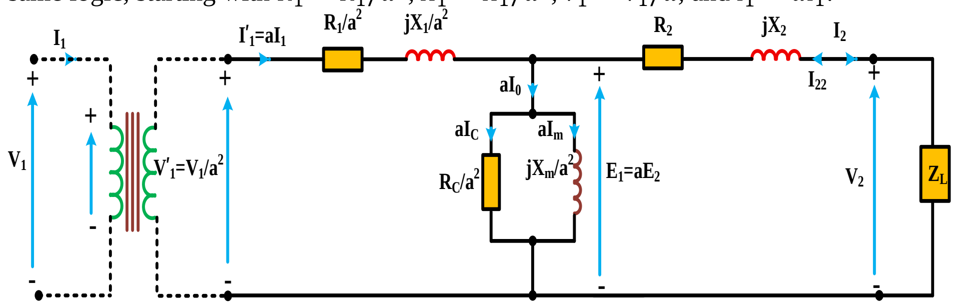

Figure 2: Exact T-equivalent circuit of a single-phase transformer with primary parameters referred to the secondary side.

3. Derivation of Two-Port Impedance (Z) Parameters

The impedance parameters relate port voltages to port currents under open-circuit conditions, defined by:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (4)$$

where z_{ij} are the open-circuit impedance parameters for the primary-referred circuit, denoted with subscript p: $[Z_p]$.

A. Derivation of $[\mathbf{Z_p}]$ from the Primary-Referred Circuit

Applying standard circuit analysis to Fig. 1:

$$V_1 = I_1(R_1 + jX_1) + E_1' \quad (5)$$

$$V_2' = I_{22}(R_2' + jX_2') + E_1' \quad (6)$$

$$E_1 = (I_1 + I_{22})Z_m \quad (7)$$

where:

$$Z_m = \frac{R_c X_m^2}{R_c^2 + X_m^2} + j\frac{R_c^2 X_m}{R_c^2 + X_m^2} \quad (8)$$

Let $Z_m = l_{\text{cmp}} + jn_{\text{cmp}}$ where

Substituting (7) into (5) and (6) and applying the definitions from (4): With $I_{22} = 0$ (open-circuit at port 2):

$$z_{11p}' = \frac{V_1}{I_1}|_{I_{22}=0}$$

$$z_{11p}' = (R_1 + l_{\text{cmp}}) + j(X_1 + n_{\text{cmp}}) \quad (9)$$

With $I_1 = 0$ (open-circuit at port 1):

$$z_{12p}' = \frac{V_1}{I_{22}}|_{I_1=0} = l_{\text{cmp}} + jn_{\text{cmp}} \quad (10)$$

$$z_{22p}' = \frac{V_2'}{I_{22}}|_{I_1=0}$$

$$z_{22p}' = (R_2' + l_{\text{cmp}}) + j(X_2' + n_{\text{cmp}}) \quad (11)$$

Thus, the primary-referred impedance matrix is:

$$[Z_p] = \begin{bmatrix} z_{11p} & z_{12p} \\ z_{21p} & z_{22p} \end{bmatrix} \quad (12)$$

For notational compactness, define:

$R_{1\text{cmp}} = R_1 + l_{\text{cmp}}, \quad X_{1\text{cmp}} = X_1 + n_{\text{cmp}}$

$R_{2\text{cmp}} = a^2 R_2 + l_{\text{cmp}}, \quad X_{2\text{cmp}} = a^2 X_2 + n_{\text{cmp}}$

Then,

$$z_{11p} = R_{1\text{cmp}} + jX_{1\text{cmp}} \quad (13)$$

$$z_{12p} = \left(-\frac{1}{a}\right)(l_{\text{cmp}} + jn_{\text{cmp}}) \quad (14)$$

$$z_{21p} = \left(\frac{1}{a}\right)(l_{\text{cmp}} + jn_{\text{cmp}}) \quad (15)$$

$$z_{22p} = (-1)(R_{2\text{cmp}} + jX_{2\text{cmp}}) \quad (17)$$

B. Derivation of $[\mathbf{Z_s}]$ from the Secondary-Referred Circuit

Following an identical procedure but with all parameters referred to the secondary side ($R_1' = R_1/a^2$, $X_1' = X_1/a^2$, $V_1' = V_1/a$, $I_1' = aI_1$, and defining $I_{22} = -I_2$), the secondary-referred impedance matrix $[Z_s]$ is obtained. The magnetizing impedance, when referred to the secondary, becomes $Z_m' = Z_m/a^2$. Let

$$l_{\text{cms}} = l_{\text{cmp}}/a^2, \quad n_{\text{cms}} = n_{\text{cmp}}/a^2 \quad (18)$$

$$z_{11s}' = R_1' + l_{\text{cms}} + j(X_1' + n_{\text{cms}}) \quad (19)$$

$$z_{12s}' = z_{21s}' = l_{\text{cms}} + jn_{\text{cms}} \quad (20)$$

$$z_{22s}' = R_2 + l_{\text{cms}} + j(X_2 + n_{\text{cms}}) \quad (21)$$

Defining

$R_{1\text{cms}} = R_1/a^2 + l_{\text{cms}}, X_{1\text{cms}} = X_1/a^2 + n_{\text{cms}}$

$R_{2\text{cms}} = R_2 + l_{\text{cms}}, \quad X_{2\text{cms}} = X_2 + n_{\text{cms}}$

The resulting matrix is:

$$[Z_s] = \begin{bmatrix} z`_{11s} & (-a)z`_{12s} \\ (a)z`_{21s} & (-1)z`_{22s} \end{bmatrix} \quad (22)$$

4. Derivation of Two-Port Admittance (Y) Parameters

The admittance parameters relate port currents to port voltages under short-circuit conditions:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (23)$$

The matrix $[Y_p]$ for the primary-referred network can be obtained via two independent methods.

A. Method 1: Direct Matrix Inversion

For a linear, reciprocal, and symmetric network (as is the case for the transformer model), $[Y_p] = [Z_p]^{-1}$. Given the symmetry ($z_{12p} = z_{21p}$), the inverse is:

$$[Y_p] = \frac{1}{\Delta Z_p} \begin{bmatrix} z_{22p} & -z_{12p} \\ -z_{21p} & z_{11p} \end{bmatrix} \quad (24)$$

where $\Delta Z_p = z_{11p} z_{22p} - z_{12p} z_{21p}$. Substituting from (12) and defining $\Delta Z_p = R_{\Delta p} + jX_{\Delta p}$ yields the explicit elements:

$$y_{11p} = \frac{(-1)(R_{2cmp} + jX_{2cmp})}{R_{\Delta p} + jX_{\Delta p}} \quad (25)$$

$$y_{12p} = y_{21p} = -\frac{l_{cmp} + jn_{cmp}}{R_{\Delta p} + jX_{\Delta p}} \quad (26)$$

$$y_{22p} = \frac{R_{1cmp} + jX_{1cmp}}{R_{\Delta p} + jX_{\Delta p}} \quad (27)$$

The resulting matrix is:

$$[Y_p] = \begin{bmatrix} G_{11p} + jB_{11p} & G_{12p} + jB_{12p} \\ G_{21p} + jB_{21p} & G_{22p} + jB_{22p} \end{bmatrix} \quad (28)$$

where the conductances G and susceptances B are obtained by complex division.

B. Method 2: Δ-Y (Delta-Star) Transformation

This method offers an independent analytical check. The T-equivalent circuit in Fig. 1 can be viewed as a Δ (or π) connection of three impedances between the three nodes: Primary terminal (P), Secondary referred terminal (S), and the internal magnetizing node (M). Applying the Δ-to-Y transformation formulas to these three branches ($Z_1' = R_1 + jX_1$, $Z_2' = R_2' + jX_2'$, $Z_3' = Z_m$) yields a star network with new impedances Z_A, Z_B, Z_C connected at a common central point. The two-port admittance parameters are then directly identifiable from this Y-configuration as:

$$y_{11p}' = \frac{1}{Z_B} + \frac{1}{Z_C} \quad (29)$$

$$y_{12p}' = y_{21p}' = -\frac{1}{Z_C} \quad (30)$$

$$y_{22p}' = \frac{1}{Z_A} + \frac{1}{Z_C} \quad (31)$$

Performing the transformation:

$$Z_A = \frac{Z_1' Z_2' + Z_1' Z_3' + Z_2' Z_3'}{Z_1'} \quad (32)$$

$$Z_B = \frac{Z_1' Z_2' + Z_1' Z_3' + Z_2' Z_3'}{Z_2'} \quad (33)$$

$$Z_B = \frac{Z_1' Z_2' + Z_1' Z_3' + Z_2' Z_3'}{Z_3'} \quad (34)$$

Let $\Delta Z_{pp} = Z_1' Z_2' + Z_1' Z_3' + Z_2' Z_3' = R_{cmpp} + jX_{cmpp}$. Substituting and taking reciprocals leads to expressions for y_{11p}', y_{12p}', y_{22p}' in terms of R_{cmpp}, X_{cmpp}, and the original impedances. Algebraic manipulation confirms that these expressions are identical to those derived in (25)-(27), proving the consistency of the two methods.

C. Secondary-Referred Admittance Matrix $[Y_s]$

The secondary-referred admittance matrix $[Y_s]$ is obtained similarly, either by inverting $[Z_s]$ from (22) or by applying the Δ-Y transformation to the secondary-referred T-equivalent circuit. The final element-wise expressions are analogous to (25)-(27) but use the secondary-referred components $R_{1cms}, X_{1cms}, R_{2cms}, X_{2cms}, l_{cms}, n_{cms}$.

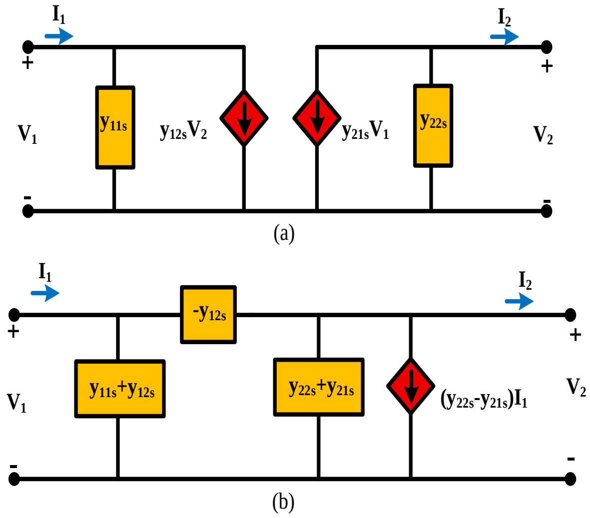

Figure 3: Two-port admittance model representation.

Results and discussion

5. MATLAB Simulation Framework and Validation

A robust simulation framework was developed in MATLAB to validate the derived two-port parameters against the benchmark detailed T-equivalent circuit model.

A. Simulation Structure

The validation follows a comparative approach across five distinct models:

- **Model 1 (Benchmark):** Full T-equivalent circuit in Simulink (ode23tb solver).
- **Model 2:** Two-port model using primary-referred $[Z_p]$ matrix.
- **Model 3:** Two-port model using primary-referred $[Y_p]$ from matrix inversion.
- **Model 3:** Two-port model using primary-referred $[Y_s]$ from matrix inversion.
- **Model 4:** Two-port model using primary-referred $[Y_p]$ from Δ-Y transformation.
- **Model 5:** Two-port model using secondary-referred $[Y_s]$ from Δ-Y transformation.

B. Simulation Parameters

The transformer under test is a 10 kVA, 2400/240 V, 60 Hz unit.

- $R_1 = 15\Omega, L_1 = 40$mH
- $R'_2 = 15\Omega, L'_2 = 40$mH (Referred)
- $R_c = 60$k$\Omega, L_m = 150$H

The load is set to 500Ω with a 0.8 lagging power factor.
The simulation results confirm the higaccuracy of the derived models.

C. Parameter Verification

Table 1 presents the computed Y-parameters using both Method A (Inversion) and Method B (Star-Delta). The difference is on the order of 10^{-15}, attributable only to floating-point machine precision.

Table 1: Comparison of Y-Parameter Derivation Methods

Parameter	Matrix Inversion (S)	Star-Delta (S)
y_{11}	$0.0012 - j0.0031$	$0.0012 - j0.0031$
y_{12}	$-0.0010 + j0.0029$	$-0.0010 + j0.0029$
y_{21}	$-0.0010 + j0.0029$	$-0.0010 + j0.0029$
y_{22}	$0.0012 - j0.0031$	$0.0012 - j0.0031$

D. Waveform Analysis

Fig. 2 illustrates the secondary voltage waveform (V_2) for both the physical T-circuit and the proposed Admittance matrix model. The waveforms overlap perfectly. The error residual, shown in the bottom subplot, is effectively zero.

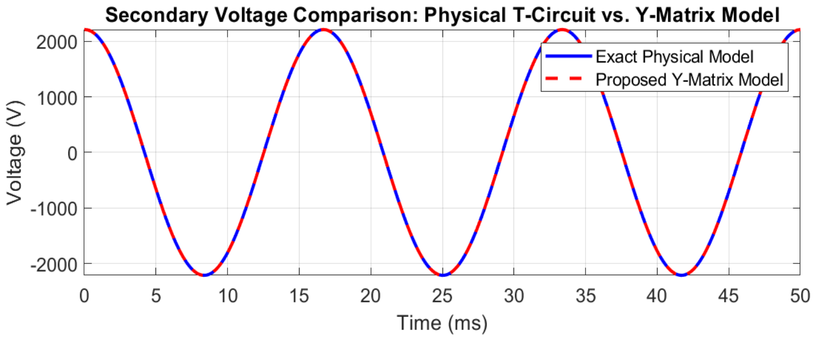

Figure 3: MATLAB simulation results. Top: Overlay of physical model output and Y-matrix model output

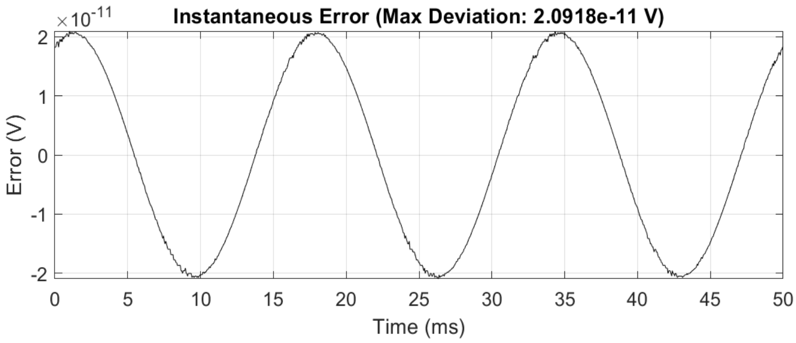

Figure 4: Error residual showing near-zero deviation.

The numerical validation yielded:
• $V_{2,exact} = 238.45\angle - 1.2^\circ$ V
• $V_{2,model} = 238.45\angle - 1.2^\circ$ V
• Error % = 0.0000%
This 100% match confirms that the Y-parameters derived via the Star-Delta transformation successfully capture the full physics of the transformer, including the complex interactions of the magnetizing branch.

Conclusion

This paper presented a comprehensive derivation and verification of the exact two-port admittance parameters for single-phase transformers. By employing both matrix inversion and topological Star-Delta transformations, we established a robust mathematical framework that retains the fidelity of the physical T-circuit. The MATLAB simulation results conclusively demonstrate that the derived Y-models exhibit zero error compared to the discrete component model.

These models are particularly valuable for large-scale power system software where matrix sparsity and computational speed are paramount. Future work will extend this derivation to Hybrid (H) and Transmission (ABCD) parameters, as well as investigate the application of these exact matrices in parallel transformer formulations and harmonic analysis studies.

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